شغوف

Calculus 2

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Reference: Varberg, Purcell, Rigdon (ninth edition) . *Calculus*.

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1. $D_x \int_0^{3x} t \cos t \, dt$

$a.$ $x \sin(3x)$

$b.$ $9x \cos(3x)$

$c.$ $x \cos(3x)$

$d.$ $-x \sin(3x)$

$$D_x \int_0^{3x} t \cos t \, dt = 3x \cos(3x)\,(3) = 9x \cos(3x)$$

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$$2.\ D_x \int_0^x t \cos t \, dt$$

$a.$ $x \sin(x) - \sin(x)$

$b.$ $-x \sin(x)$

$c.$ $x \sin(x)$

$d.$ $x \cos(x)$

$$D_x \int_0^x t \cos t \, dt = x \cos(x) = x \cos(x)$$

3. The area of the region R under the curve $y = \cos x$, between $x = 0$ and $x = \frac{\pi}{2}$ is:

a. 2

b. 7

c. $\frac{23}{3}$

d. 1

x	0	$\pi/2$
y	1	0

$$\int_0^{\frac{\pi}{2}} \cos x \, dx = \sin x \Big|_0^{\frac{\pi}{2}}$$

$$= \sin\frac{\pi}{2} - \sin 0 = 1 - 0 = 1$$

4. The area of the region R under the curve $y = 3x^2 - 5$, between $x = 0$ and $x = 1$ is:

a. 4

b. 1

c. 2

d. -4

x	0	1
y	-5	-2

$$-\int_0^1 3x^2 - 5\,dx = -\frac{3x^3}{3} + 5x\Big|_0^1$$

$$= -x^3 + 5x\Big|_0^1 = (-1+5) - (0+0) = 4$$

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$$5.\ D_x \ln\left(\frac{2x+1}{x-1}\right)$$

a. $\dfrac{1}{2x+1} - \dfrac{1}{x-1}$

b. $\dfrac{x}{2x+1} + \dfrac{1}{x-1}$

c. $\dfrac{2}{2x+1} - \dfrac{1}{x-1}$

d. $\dfrac{x}{2x+1} - \dfrac{1}{x-1}$

$$D_x(\ln(2x+1) - \ln(x-1))$$

$$= \frac{1}{2x+1}(2) - \frac{1}{x-1}$$

$$= \frac{2}{2x+1} - \frac{1}{x-1}$$

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$$6. \int_3^5 \frac{1}{x-3}\,dx$$

$a.$ ∞

$b.$ 1

$c.$ 2

$d.$ π

$$u = x - 3, \qquad du = dx$$

$$= \ln|x-3|\Big|_3^5 = \ln|5-3| - \ln|3-3|$$

$$= \ln 2 - \ln 0 = \ln 2 + \infty = \infty$$

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7. $\frac{\ln(8)}{\ln(2)}$

$a.$ 7

$b.$ 1

$c.$ 4

$d.$ 3

$$\frac{\ln 8}{\ln 2} = 3$$

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8. The inverse function of $f(x) = (x-3)^{\frac{1}{4}}$

a. $f^{-1}(x) = (x-3)^4$

b. $f^{-1}(x) = (x-3)^4$

c. $f^{-1}(x) = x^4 + 3$

d. $f^{-1}(x) = x^4 - 3$

$$y = (x-3)^{\frac{1}{4}}$$

$$y^4 = x - 3$$

$$x = y^4 + 3$$

$$f^{-1}(y) = y^4 + 3$$

$$f^{-1}(x) = x^4 + 3$$

9. Let $f(x) = x^3 + 5x$, then $(f^{-1})'(6)$

a. $\frac{1}{32}$

b. $-\frac{1}{32}$

c. $-\frac{1}{8}$

d. $\frac{1}{8}$

$[1, 6]$

$f(x) = x^3 + 5x$

$f'(x) = 3x^2 + 5$

$f'(1) = 3 + 5 = 8$

$(f^{-1})'(6) = \frac{1}{f'(1)} = \frac{1}{8}$

10. $e^{x \ln y}$

$a.$ y^x

$b.$ $x+y$

$c.$ $x-y$

$d.$ xy

$$e^{x \ln y} = e^{\ln y^x} = y^x$$

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$$11. \int_0^1 e^{-x}\, dx$$

$a.$ e

$b.$ $1 - \frac{1}{e}$

$c.$ $e + 1$

$d.$ $e - 1$

$$= -\int_0^1 -e^{-x}\, dx = -e^{-x}\Big|_0^1 = e^{-x}\Big|_1^0$$

$$= e^0 - e^{-1} = 1 - \frac{1}{e}$$

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12. $D_x 5^{\tan^{-1}(x)}$

$a.$ $5^{\tan^{-1} x}$

$b.$ $\dfrac{5^{\tan^{-1} x} \ln 5}{x^2+1}$

$c.$ $5^{\tan^{-1} x} \ln 5$

$d.$ $5^{\tan^{-1} x} \sec^2 x$

$$= 5^{\tan^{-1} x} \ln 5 \left(\frac{1}{x^2+1}\right)$$

$$= \frac{5^{\tan^{-1} x} \ln 5}{x^2+1}$$

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13. $\int 2^{3x}\,dx$

a. $\dfrac{2^{3x}}{3\ln 2} + c$

b. $2^{3x} + c$

c. $3(2^{3x}) + c$

d. 2^3

$$= \frac{1}{3}\int 2^{3x}\,3\,dx = \frac{1}{3}\frac{2^{3x}}{\ln 2} + c$$

14. $D_x \log_3 \sin x$

$a.$ $3\cos x$

$b.$ $\dfrac{\cos x}{\sin x \ln 3}$

$c.$ $\cot x$

$d.$ $3\ln(\cos x)$

$$= \frac{1}{\sin x \ln 3}\cos x = \frac{\cos x}{\sin x \ln 3} = \frac{\cot x}{\ln 3}$$

$15. \sin^{-1}(-1)$

$a.$ $\frac{-\pi}{2}$

$b.$ 1

$c.$ π

$d.$ $-\pi$

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16. $\tan^{-1}(-1)$

$a.$ π

$b.$ 2π

$c.$ $\frac{\pi}{4}$

$d.$ $-\frac{\pi}{4}$

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17. $D_x \sin^{-1}(x^2)$

a. $\dfrac{2x}{\sqrt{1-x^4}}$

b. $\dfrac{6x}{\sqrt{1-9x^4}}$

c. $\dfrac{x}{\sqrt{1-x^2}}$

d. $\dfrac{-x}{\sqrt{1-3x^2}}$

$$= \frac{1}{\sqrt{1-(x^2)^2}} . 2x$$

$$= \frac{2x}{\sqrt{1-x^4}}$$

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18. $D_x \tan^{-1}(e^x)$

a. $\dfrac{e^x}{1+e^{2x}}$

b. $\dfrac{e^x}{\sqrt{1-e^x}}$

c. $\dfrac{1}{\sqrt{1-e^x}}$

d. $\dfrac{1}{1+e^{2x}}$

$$= \frac{1}{1+(e^x)^2} . e^x = \frac{e^x}{1+e^{2x}}$$

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19. $\int_{0}^{1} \frac{1}{\sqrt{4-x^2}} dx$

$a.$ π

$b.$ $\frac{\pi}{6}$

$c.$ 2

$d.$ $\frac{\pi}{2}$

$$\int_{0}^{1} \frac{1}{\sqrt{2^2-x^2}} dx = \sin^{-1}\frac{x}{2}\Big|_{0}^{1}$$

$$= \sin^{-1}\frac{1}{2} - \sin^{-1} 0 = \frac{\pi}{6} - 0 = \frac{\pi}{6}$$

شغوف

20. $\int \frac{e^x}{1+e^{2x}}\,dx$

a. $\ln|e^x+1|+c$

b. $\frac{e^x}{e^x-x}+c$

c. $\tan^{-1}(e^x)+c$

d. $\ln|e^x+x|+c$

$u=e^x, du=e^x dx$

$$\int \frac{e^x}{1+(e^x)^2}\,dx=\int \frac{1}{1+u^2}\,du$$

$$=\tan^{-1}u+c=\tan^{-1}(e^x)+c$$