(1)

King Saud University

Department of Mathematics

2nd Semester 1438-1439 H

MATH 244 (Linear Algebra)

Final Exam

Duration: three hours

Name:

Sequence Number:

Teacher's Name:

Section:

Note: The exam consists of 8 pages

Question	Mark
Question I	
Question II	
Question III	
Question IV	
Question V	
Question VI	
Bonus	
Total	

Question Number	1	2	3	4	5	6	7	8	9	10
Answer	a	a								

Question I

Choose the correct answer, then fill in the table above:

(1) The values of k that makes the linear system $\begin{matrix} 9x + ky = a \\ kx + y = b \end{matrix}$ consistent for every $a, b \in \mathbb{R}$, are:

(a) $\{-3, 3\}$ (b) $\mathbb{R} - \{3\}$ (c) $\mathbb{R} - \{-3, 3\}$ (d) None of the previous

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(2) If $u = (3, 1)$ and $v = (-1, 4)$, then $||-2u|| + ||2v|| =$

(a) $2(\sqrt{10} + \sqrt{17})$ (b) $2(-\sqrt{10} + \sqrt{17})$ (c) $-2(\sqrt{10} + \sqrt{17})$ (d) None of the previous

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(3) If A is the zero matrix and $T_A: \mathbb{R}^n \to \mathbb{R}^m$ is the corresponding matrix operator, then the column space of A is:

(a) $\mathbb{R}^n$ (b) $\mathbb{R}^m$ (c) $\{0\}$ (d) None of the previous

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(4) If T_1 is the reflection operator about the line $y = x$ in $\mathbb{R}^2$, and T_2 is the orthogonal projection on the y-axis, then $[T_1 \circ T_2]$ is:

(a) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$ (d) None of the previous

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(5) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $AB = \begin{bmatrix} 7 & 3 & -4 \\ -1 & 6 & 21 \\ 9 & 0 & 5 \end{bmatrix}$ then the second column vector of B is:

(a) $\begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 3 \\ 12 \\ 0 \end{bmatrix}$ (d) None of the previous

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(6) Let $f_1 = e^x, f_2 = x$ and $f_3 = 5$. Then the Wronskian of f_1, f_2 and f_3 is:

(a) $5e^x$ (b) $-5e^x$ (c) $(5 - x)e^x$ (d) None of the previous

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(7) If A is a 5×7 matrix and $rank\ A = 3$, then the nullity of A^T equals:

(a) 2 (b) 3 (c) 4 (d) None of the previous

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(8) The dimension of the set $\boldsymbol{Span}\{(\mathbf{2},\mathbf{0}),(\mathbf{1},\mathbf{1})\}$ equals:

(a) $\mathbf{0}$ (b) $\mathbf{1}$ (c) $\mathbf{2}$ (d) None of the previous

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(9) If $\boldsymbol{u},\boldsymbol{v} \in \mathbb{R}^3$ and $\boldsymbol{u},\boldsymbol{v}$ are orthogonal vectors, then $\boldsymbol{u}\cdot\boldsymbol{v} =$

(a) $\mathbf{0}$ (b) $(\mathbf{0},\mathbf{0},\mathbf{0})$ (c) $\mathbf{1}$ (d) None of the previous

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(10) The eigenvalues of $\boldsymbol{A}$, where $\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 5 \\ 0 & 0 & -1 \end{bmatrix}$ are:

(a) $\mathbf{1}$ and $\mathbf{2}$. (b) $\mathbf{1}$ and $-\mathbf{1}$. (c) $\mathbf{1}$, $\mathbf{2}$, and $-\mathbf{1}$. (d) None of the previous

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Question II

If $\boldsymbol{W} = \{(\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{d}) \in \mathbb{R}^4 \mid \boldsymbol{d} = \boldsymbol{a}+\boldsymbol{b},\ \boldsymbol{c} = \boldsymbol{a}-\boldsymbol{b}\}$. Then answer the following:

1. Prove that $\boldsymbol{W}$ is a subspace of $\mathbb{R}^4$.

2. Find a basis for $\boldsymbol{W}$.

Question III

1. Show that the polynomials $\boldsymbol{p_1(x) = x^2 + 1}$, $\boldsymbol{p_2(x) = x^2 - 1}$, and $\boldsymbol{p_3(x) = 2x + 1}$ form a basis for $\boldsymbol{P_2}$.

2. Find the coordinate vector $(\boldsymbol{p})_{\boldsymbol{S}}$ of $\boldsymbol{p(x) = x^2 + x - 1}$ relative to the basis $\boldsymbol{S} = \{\boldsymbol{p_1}, \boldsymbol{p_2}, \boldsymbol{p_3}\}$.

Question IV

Let $A = \begin{bmatrix} 3 & -6 & 0 & 9 \\ 1 & -1 & 2 & 2 \\ 3 & 1 & 4 & 12 \\ -2 & 5 & 1 & -6 \end{bmatrix}$. Then answer the following:

1. Find a basis for the row space and the column space of the matrix A.

2. Find the rank and the nullity of A.

3. Find a basis for the solution space of $Ax = 0$.

Question V

Let $T: R^2 \rightarrow R^2$, be the transformation defined by $T(x, y) = (-x + y\,,\ x + y)$. Then answer the following:

1. Find the matrix $[T]$.

2. Prove that T is one-to-one.

3. Find $T^{-1}(x, y)$.

Question VI

Let $\boldsymbol{A} = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$. Then answer the following:

1. Prove that $\boldsymbol{\lambda} = \mathbf{2}$ and $\boldsymbol{\lambda} = \mathbf{4}$ are eigenvalues of $\boldsymbol{A}$.

2. Find a basis for the eigenspace corresponding to the eigenvalue $\boldsymbol{\lambda} = \mathbf{2}$.